

## WANTED: CHRISTIAN PERSPECTIVES IN THE PHILOSOPHY OF MATHEMATICS

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Whatever has Christianity to do with mathematics? This is an old chestnut; and it has become customary to say that, while one's faith affects his general attitude to both subject matter and students, it has no further and no distinctive bearing on mathematics. Whatever one might say about a Christian psychology or a Christian philosophy or even a Christian mathematician, the idea of a Christian mathematics seems vacuous.

Professor Perciante however dares to ask whether the ontological problems posed by mathematics might profitably be addressed from within a Christian world-view that has been philosophically developed. For while  $3 + 5 = 8$  in a base 10 number system whether the mathematician is Christian, Hindu or Marxist, yet the assumptions a mathematician makes regarding truth and meaning and what kinds of reality exist do indeed have ramifications for Christian thought. If not Christian mathematics, then perhaps a Christian philosophy of mathematics?

Professor Spradley makes a related point. Science can no longer claim to exercise independent and autonomous reasoning of a purely logical sort, for observation is theory-laden and theories reflect larger paradigms shaped by a variety of human considerations well known to the sociology of knowledge. If the notion of a purely empirical and presuppositionless science is an empty will-o'-the-wisp, so also may be the notion of a presuppositionless mathematics.

No less a figure than Nelson Goodman applies the same supposition to logic.

It is often taken for granted that everything customarily called logic ... is purely neutral machinery that can be used without ontological implication in any constructional system. But this neutrality is preserved only so long as the machinery is uninterpreted. If we use variables that we construe as having entities of any given kind as values, we acknowledge that there are such entities ... If we use variables that call for classes as values, we acknowledge that there are [real] classes [as well as individuals].<sup>1</sup>

So then we are faced with at least the possibility that there is no presuppositionless science, no presuppositionless logic, and no presuppositionless mathematics. Perhaps a Christian world-view does make some difference to mathematics after all.

In a recent series of lectures to teachers in Reformed colleges, published under the title Reason within the Bounds of Religion<sup>2</sup>, Nicholas Wolterstorff of Calvin College has addressed the same issue. After discussing the role of the "control beliefs" inherent in any world-view in the formation and criticism of theory in any discipline, he calls for Christian scholars to explore theoretical possibilities that might be suggested, although not necessarily implied in a formal logical sense by Christian control beliefs remembering all the time that the same theoretical possibilities might well be acceptable within a variety of differing world-views. And with specific regard to mathematics, Herman Dooyeweerd remarks that the laws and structures that govern mathematics are common to all men, whether or not Christian, only because mathematical reasoning is something created by God. It finds its basis and its meaning, just as any other science does in the law structure given to creation by the creator-God.<sup>3</sup> Neither man nor mathematics nor any other created thing functions with

complete autonomy as a law unto itself.

One way in which philosophy can help is in conceptualizing the creation order or law structure that makes mathematics or any other discipline possible, that is to say in articulating the ontological precommitments of a discipline. The most influential way of doing this in both the past and the present employs theories of universals such as Plato initiated in his attempt to explain rational thought and the intelligible order of nature. It should be kept in mind that while theories of universals have also been applied in theology and ethics and elsewhere, they arose initially in order to handle ontological questions posed by Greek cosmology, and gained considerable initial impetus from a mathematician, Pythagoras, with his conception of the metaphysical significance of numbers. Since then, they have repeatedly been addressed to science as well -- see for example, Whitehead's discussion of "laws of nature" in his Adventures of Ideas. In fact such theories bear on every area of thought and every sphere of existence in nature and in man. This most general kind of theory is of course part of what distinguishes philosophy from any particular science.

The pertinence of this subject to mathematics is obvious from attempts to identify theories about the foundations of mathematics with various theories of universals.<sup>4</sup> Specifically it is suggested that the logicism of Frege and Russell reflects Platonic realism, that Hilbert's formalism is nominalistic, and that Brouwer's intuitionism is analogous to the kind of conceptualism we find in Kant.

I shall therefore describe briefly these three types of theory, beginning in each case with a classic historical formulation and moving to its restatement in recent analytic philosophy. Then I shall suggest ways in which Christian perspectives bear on theories of universals and so on mathematics. But the outcome will prove far from unambiguous.

Realism – sometimes called Platonism, although other realisms have been developed than Plato's own version – is the theory that abstract entities exist independently of the mind that thinks them. There are two elements of importance here: (a) a world of extra-mental realities that both is and is what it is quite apart from whether any of us know it, and (b) the notion of an abstract entity as distinct from either an observable resemblance between individual things, on the one hand, or a general term, even a variable, on the other hand. Abstract entities are extra-mental realities in their own right, not the numerical or verbal: symbols we use to denote them and not just general class concepts. In addition, therefore, to three blind mice, three musketeers, three major prophets, three symposiasts and three branches of government, reality contains an abstract entity called "three" to which these and other sets of three are somehow related. Numbers are abstract entities of this sort, so that mathematics is about real relationships between real entities, not just about man-made symbols, man-made rules of inference, or even the space-time world.

The early twentieth century saw a revival of realism in European phenomenology: in Meinong's Gegenstandstheorie, for instance, and in Husserl's attempt to establish thereby the foundations of logic, mathematics and science. British and American philosophy, not entirely independent of the European influence, developed its own version of this realism. Philosophers like G.E. Moore, Bertrand Russell and A.N. Whitehead were well to the fore. Often a distinction was made between the existence of physical objects and the subsistence that logical and mathematical objects like universals enjoy. The work of Frege, along with the Principia Mathematica of Russell and Whitehead, should be seen in this context.

Realistic theories run into logical problems of various sorts. For instance: (1) How are these two kinds of reality (universals and particulars) related? In Plato this was known as the problem of participation. (2) Do universals afford any logical economy? There

seemingly need to be as many as or more than there are particulars. (3) How do we know either that they are or what they are: the epistemological problem.

A modified realism, which Nicholas Wolterstorff develops in his book On Universals, has lately attracted attention in philosophical circles. Rather than regarding universals as themselves subsistent entities he conceives of them as "kinds of possibles," logical possibilities for any possible world, whose possibility is independent of whether or not we think them. They are then an objective order of extra-mental possibilities, only some of which are exemplified in our particular world. Reality accordingly consists of universals, exemplifications and cases. The universal possibility of wisdom, for instance in the case of Socrates' wisdom, is exemplified by Socrates. "Three" is exemplified by mice in the case of three blind mice and by branches of government in the case of our executive, judiciary and legislature, and by Perciante, Spradley and Holmes in the case of this symposium. On this view universals are not immaterial entities, but they are nonetheless objective, extra-mental and real: they are real logical possibilities. A square circle would be excluded from any possible world, logical, mathematical or physical, but the cube root of 27 is the same for any possible world. Wolterstorff is currently trying to apply his theory of possibles to provide an objective basis for aesthetics.

Nominalists and conceptualists are not likely to be satisfied.

Nominalism denies both the extra-mental reality of universals and the notion that there are such logical concepts within the mind. Only particulars exist and only particulars can be thought or spoken of. We entertain no abstract ideas and use no abstract terms -- only general or class terms that refer to particulars. Logical economy, in the person of Occam's razor, simply refuses to introduce logically superfluous entities; and since both real universals and universal concepts are logically superfluous, the nominalist goes without.

The logical economy argument is reinforced epistemologically. In the eighteenth century, George Berkeley found no empirical basis for any abstract ideas, especially not the Newtonian ideas of absolute space and time, of matter as a substratum or of causal force or power. These are not empirical concepts, generalized from our observations, but abstract ideas devoid of empirical reference and devoid of extra-mental reality. In this empiricist tradition, aided by David Hume and the positivists, mathematics becomes either a descriptive, empirical science (as in J.S. Mill) or else a purely formal discipline dealing only with mathematical symbols and not with extra-mental realities. As such it may or may not have useful application.

One recent example is the modified nominalism of Nelson Goodman who states the principle of logical economy thus: "no distinction of entities without distinction of content." This he regards as an important stipulation for a sound philosophical system. Consequently he refuses to allow not only the extra-mental reality of abstract entities, but also any language of abstract entities. Talk of a class as a different kind of entity than the members of the class is therefore taboo. There are composite individuals like a particular government with three branches or a particular group of three blind mice or symposiasts, but there are in reality only particular entities, be they simple or complex.<sup>6</sup>

Others are equally explicit, even if not quite as emphatic. Rudolph Carnap admits that using the language of abstract entities does not require accepting the reality of abstract entities. But the choice between nominalist and realistic languages is one of utility: which language is most practicable for doing what we want. This is like saying that the choice between a Ptolemaic and a Copernican cosmology is nothing but a matter of operational simplicity: nothing about reality is at stake. He elaborates his claim by distinguishing internal from external questions about a language. The internal question "Is there a prime

number greater than 100?" can be answered by simply using the rules of prime number language. It is an analytic question. But the question, "Do prime numbers really exist?" is an external question that cannot be answered either analytically within the language or empirically (like questions of utility can). As a good empiricist, Carnap believes that any meaningful statement must be either analytic or empirical. The existence question is neither: hence it is cognitively meaningless.<sup>7</sup>

W.V.O. Quine should be mentioned here, because he too objects on the principle of logical economy to the overpopulated universe of realists. To make his point he distinguishes between meaning and naming. The latter is referential, so that a name has logical extension, which means that the thing named really exists, whether it be the mythical Pegasus, the cube root of 27, or the "three" in which men and mice and governments all have a share. But in contrast to naming, "meaning" has to do with logical intention, and is just about a linguistic or logical system. Words and symbols do not have a meaning, whether particulars or universals. Words and symbols are however translatable into other logically equivalent symbols. When we give them extra-mental or extra-linguistic reference, and treat them as names, we are attempting to round out or simplify our linguistic scheme by means of postulates that add to the logical usefulness of the scheme. We should however "adopt the simplest conceptual scheme, into which the disordered fragments of our experience can be fitted and arranged,"<sup>8</sup> and that is definitely not realism.

The parallel to mathematical formalism is plain. Numbers are symbols, axioms are postulates, and mathematics is the logical analysis of their interrelationships without concern over the question of existence. A mathematical language is quite self-contained and autonomous.

Conceptualism is the apparent cohort of mathematical intuitionism. The conceptualist agrees with nominalism in rejecting the extra-mental reality of universals, but disagrees about the purely analytic and formal nature of mathematics. With the realist, he is convinced that we do traffic in universal conceptions. Immanuel Kant provides two points of reference:

- a) Mathematical judgments are not analytic (as in nominalism) but are synthetic *a priori*: that is, while they add to our knowledge as all synthetic judgments do, they are about universal concepts that the mind itself contributes to human thought. According to Kant, arithmetic presupposes the concept of time, and geometry, the concept of space. We realize now of course, that Kant's space and time were Newtonian concepts and not the universal and necessary principles he thought them to be. So the intuitionist proposes instead that the mind itself creates the mathematical concepts with which it deals, constructing a systematic scheme to elaborate them more and more fully.
- b) Kant found that his universal concepts lead to such logical antinomies that no firm conclusion is possible about extra-mental realities. Universal concepts structure our perception of and thinking about experience (they have useful applications) but they tell us nothing about reality. Hence Kant's famous phenomenal-noumenal distinction. And the intuitionist concurs in refusing arbitrarily to ascribe extra-mental reality to his mathematical concepts.

Now how does one choose between these conflicting kinds of theory? Not merely by means of their logical simplicity, I suggest, but by judging which theory best answers the ontological problems to which they are addressed. The choice is not, I suggest, simply a matter of finding the simplest conceptual scheme, but rather of finding the simplest and most adequate conceptual scheme. And that is where Christian perspectives enter, for the

Christian seeks a conceptual scheme that adequately handles Christian intellectual concerns as well as more general philosophical and mathematical and scientific interests.

I suggest that the Christian has three stakes in this matter.

First, the recent nominalism of Carnap and Quine refuses to address the ontological question, reducing it to a purely pragmatic or semantical decision. Here arise all the philosophical problems involved in separating language from reality, and all the problems introduced by the positivist's infamous elimination of metaphysics. The Christian has a stake in metaphysics and cannot refuse the ontological question, for theism itself is a metaphysical position, and the idea of an orderly creation has ontological implications.<sup>9</sup>

Second, theories of universals, like other metaphysical theories, have theological ramifications that medieval philosophy explored in detail. A realistic theory implies that an objective and intelligible order prevails throughout all creation, of which man is part, and that God is a rational being whose creative work is governed by intelligible archetypes that even he cannot change. This poses the classic debate between intellectualist, and voluntarist conceptions of personality, both man's and God's. On the one conception, reason rules the will, order prevails over creative novelty and the universe over the individual. What then shall we say about free will, creative novelty and individuality in man? And what about the sovereign freedom of God in creating?

Understandably some Christian thinkers turned from realism to the other conception, a voluntaristic nominalism.<sup>10</sup> Individuality is then not subordinate to the universal nor novelty to order nor will to intellect. William of Occam took this path declaring that realism destroys the freedom of God which nominalism restores. Whatever God wills is chosen not to conform to the necessity of extra-mental universals, but in sovereign freedom. Stealing could even be right, if God so chose. Occam's nominalism shaped Lutheran theology. Lutheran theology pervaded the English church. The English churchman and philosopher George Berkeley adopted it in fighting the deterministic deistic implications of mechanistic science.

The Christian's second stake then is in whether the creation order is due to universal archetypes in the mind of God or to the sovereign choice of the creator. But these options may not be mutually exclusive. In fact Wolterstorff's theory of universals as logical possibilities leaves God free to choose as he will among many possible worlds and many possible creation orders. And it leaves man free to choose between various logically possible paths as well. The only thing neither God nor man can ever possibly do is the logically impossible. Not even God can draw a square circle or make  $3 + 5 = 17$  in a base 10 number system.

The third stake is in limiting the physicalist interpretation of things. One widespread view in philosophy today is that all our language and concepts about everything can be given a physicalist interpretation. Talk about minds and mental states is translatable into talk about physical states (as in Skinner's behaviorism) until it no longer is kosher to suppose that anything else exists. This smells like materialism in modern dress. But if real universals exist, then at least some things exist that are not physical, and physicalism fails. Does not the Christian have a stake in this?

Now while the stakes are high, I want to suggest that they are not decisive. With regards to the anti-metaphysical temper, first, Luther and others have been undisturbed. Theology after all is about an individual God whose particular historical acts speak redemptively to individual men. If we have theology, do we need metaphysics? I think so, but Christian nominalists do not.

Second, the will and intellect debate is by no means settled. A voluntarist view of man and God avoids talk of a universal and necessary creation order and leaves universals well enough alone. I think it should not but others disagree. Moreover, even granted a universal and intelligible creation order, is a theory of universals the only way to explain it? Perhaps some other option should be explored -- like Dooyeweerd's doctrine of law spheres. Or is that a voluntaristic nominalism all over again?

Third, the physicalist threat might also be handled in other ways. If the existence of any one non-physical entity falsifies the physicalist thesis, then something other than real universals would do the job just as well. God himself does. If God exists, then at least one non-physical being exists and physicalism is false; and other non-physical things might exist as well. Some Christian thinkers, moreover, have asked whether a physicalist interpretation of man might be logically compatible with Biblical anthropology, including the idea of a future state.<sup>12</sup> So the physicalism dispute might not be decisive either. The Christian could well have other logical moves up his sleeve than the appeal to real universals.

My point is that Christian perspectives do not settle the problem of universals and do not therefore establish a distinctive philosophy of mathematics. What they do provide is perspective -- perspective on the various ramifications of various alternatives, so that at least we know something of what we're getting ourselves into in the foundations of mathematics.

#### Footnotes

1 From The Structure of Appearances (2nd ed., Bobbs-Merrill, 1966), p. 34. Reprinted in The Problem of Universals, ed. R. J. Van Iden (Appleton-Century-Crofts, 1970), p. 229.

2 (Eerdmans, 1976).

3 A New Critique of Theoretical Thought (Presbyterian and, Reformed Publ. Co., 1960), vol. I, p. 115 f.

4 See for instance Stephen Barker, Philosophy of Mathematics (Prentice-Hall, 1964), ch. 4, and Paul Benacerraf and Hilary Putnam (eds.), Philosophy of Mathematics (Prentice-Hall, 1964), Introduction. Russell began as a realist concerning universals, but subsequently moved in a nominalistic direction.

5 (University of Chicago Press, 1970). For a general discussion of contemporary views, see Hilary Staniland, Universals (Anchor Books, 1972) and for anthologies of important materials from the debate, see R.J. Van Iden (ed.), The Problem of Universals (Appleton-Century-Crofts, 1970) and Charles Landesman (ed.) The Problem of Universals (Basic Books, 1971).

6 See The Structure of Appearance, also the selection previously, cited from R.J. Van Iden's The Problem of Universals, p. 229 ff.

7 "Empiricism, Semantics, and Ontology," in Landesman, op. cit. p. 228. Reprinted from Meaning and Necessity, (University of Chicago Press, 2nd ed., 1956).

8 "On What There Is," in From a Logical Point of View (Harper Torchbooks, 2nd ed., 1961), p. 16. See also "Logic and the Reification of Universals," Ibid., p. 102. The former

essay is reprinted in both the Van Iten and the Landesman anthologies.

- 9 On the metaphysical involvement of Christianity, see further this author's Christian Philosophy in the Twentieth Century (Craig Press, 1969), ch. 5 and Faith Seeks Understanding (Eerdmans, 1971), ch. 2. Also the article "Christian Philosophy" in Encyclopedia Britannica, 15th edition.
- 10 On voluntaristic nominalism in the Christian understanding of science see M.B. Foster's articles in Mind, LXIII (1934), 446; XLIV (1935), 439; XLV (1936), 1; entitled "The Christian Doctrine of Creation and the Rise of Modern Natural Science."
- 11 I am indebted for this observation to Professor Edward Schoen of Western Kentucky University.
- 12 See a recent article by Bruce Reichenbach of Augsburg College, "Re-Creationism and Personal Identity," Christian Scholars Review, V (1975), 326-330.